2 a) $P = (\text{total sales}) - (\text{expenses})$

$f = xyz^2 - 10000(x+y+z)$ objective

b) $g = x+y+z = 8000$ constraint 4

c) $\nabla f = \lambda \nabla g$ gives

$yz^2 - 10000 = \lambda$ 1

$xz^2 - 10000 = \lambda$ 2

$2xyz - 10000 = \lambda$ 3

from 1 and 2 $yz^2 - 10000 = xz^2 - 10000$

$z^2(y-x) = 0$

$\underbrace{z = 0 \text{ (lose money)}}_{\text{don't want}}$ $y = x$ 5

from 3 and 2 $xz^2 - 10000 = 2x^2z - 10000$

$xz(z - 2x) = 0$

$\underbrace{x = 0 \quad z = 0}_{\text{lose money on both so don't want}}$ $2x = z$ 6

Put 5 and 6 into constraint 4

$x + y + z = x + x + 2x = 4x = 8000$

$x = 2000$

$\therefore$ $x = y = 2000$ $z = 4000$

Check: $P\Big|_{2000,2000,4000} = 64\times10^{12} - 10^4(8\times10^3)$, but

at another pt on constraint, like $(4000, 2000, 2000)$,

$P\Big|_{4000,2000,2000} = 32\times10^{12} - 10^4(8\times10^3)$ which is smaller.

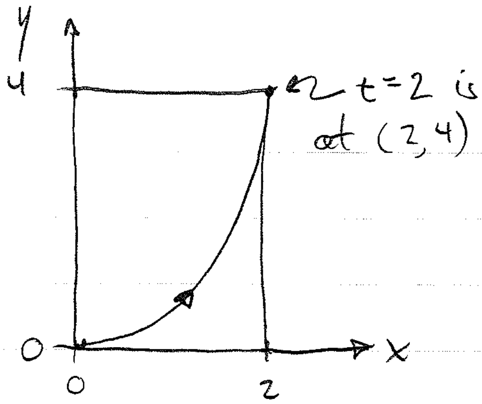
Thus we found a max.

3

$\underline{r} = t\,\hat{i} + t^2\,\hat{j}$ $\qquad T = xy$

$\underline{v} = \hat{i} + 2t\,\hat{j}$ $\qquad \nabla T = y\hat{i} + x\hat{j}$

$|\underline{v}| = \sqrt{1+4t^2}$



a) $\left.\dfrac{dT}{dt}\right|_{path} = \nabla T \circ \underline{V}$

$= (y\hat{i} + x\hat{j}) \circ (\hat{i} + 2t\hat{j})$

$= (t^2\hat{i} + t\hat{j}) \circ (\hat{i} + 2t\hat{j}) = 3t^2$ (on path $\underline{r}(t)$)

Max. of $\dfrac{dT}{dt} = 3t^2$ on path $(0 \le t \le 2)$ occurs when $t = 2$

in which case $dT/dt = 12$. ⟵

b) $\dfrac{dT}{ds} = \dfrac{dT}{dt}\,\dfrac{dt}{ds} = \dfrac{dT}{dt}\,\dfrac{1}{|\underline{V}|} = \dfrac{3t^2}{\sqrt{1+4t^2}}$

Max. of $\dfrac{dT}{ds} = \dfrac{3t^2}{\sqrt{1+4t^2}}$ for $0 \le t \le 2$ occurs when $t = 2$

in which case $dT/ds = 12/\sqrt{17}$. ⟵

c) Since $\dfrac{dT}{ds} = \nabla T \circ \hat{u}$ and ∇T depends only on field $T = xy$

and $\hat{u}$ depends only on <u>shape</u> of path

then dT/ds is indep. of speed.

However $\dfrac{dT}{dt} = \nabla T \circ \underline{V}$ and $\underline{V}$ depends on speed, so

dT/dt will change.

In fact $\dfrac{dT}{dt} = \dfrac{dT}{ds}\cdot\dfrac{ds}{dt} = \nabla T \circ \hat{u} \cdot |\underline{V}|$ indicates

that dT/dt will double if speed doubles.