1)

a)

Compute ∇f for

$$\nabla f = f_x \hat{\imath} + f_y \hat{\jmath},$$
$$= (60x - 60xy^2)\hat{\imath} + (60y - 60x^2y)\hat{\jmath}.$$

Set $\nabla f = 0$ to find critical points for

$$\begin{cases} x - xy^2 = 0 \\ y - x^2y = 0 \end{cases} \Rightarrow \begin{cases} x(1-y^2) = 0 \\ y(1-x^2) = 0 \end{cases}$$

So the critical points are $(0,0)$ & $(\pm 1, \pm 1)$.

The minimum (bottom) is at $(0,0,8000)$.

Identifying the point

Justifying

$$H = 60^2 \begin{vmatrix} 1-y^2 & -2xy \\ -2xy & 1-x^2 \end{vmatrix} = [(1-y^2)(1-x^2) - 4x^2y^2]60^2$$

So, $H|_{(0,0)} = 60^2 > 0$ & $f_{xx} = 60 > 0$ & $(0,0,8000)$ is a minumum.

b)

Identifying $(\pm 1, \pm 1)$

Justifying is also 4 pt:

$$H|_{(\pm 1, \pm 1)} = -4(60^2) < 0,$$

so they are saddle points.

c) Since $f(\pm 1, \pm 1) = 8030$ & $f(0,0) = 8000$, max -

imum depth is $8030 - 8000 = 30$.
If they wrote 8030, we marked off

Problem 5

a. $\frac{dx}{x} = .01 \qquad \frac{dy}{y} = .02 \qquad \frac{dz}{z} = .03$

(a) $\frac{df}{f} = \frac{y^2 dx}{xy^2} + \frac{2yx\,dy}{xy^2} = \frac{dx}{x} + \frac{2dy}{y} = .01 + .04 = .05$
$= \boxed{5\%}$

(b) $\frac{df}{f} = \frac{3zy^2 dy}{zy^3} + \frac{y^3 dz}{zy^3} = \frac{3dy}{y} + \frac{dz}{z} = .06 + .03$
$= .09$
$= \boxed{9\%}$

(c) $\frac{df}{f} = \frac{\frac{3x^2}{z^2} dx}{x^3/z^2} + \frac{-\frac{2x^3}{z^3} dz}{x^3/z^2} = \frac{3dx}{x} - \frac{2dz}{z}$
$= 0.03 - 0.06 = -.03$
$= \boxed{-3\%}$

b. You should pick equation (c) and divide the error of z by 2.

$\frac{df}{f} = \frac{3dx}{x} - \left(\frac{1}{2}\right)\frac{dz}{z} = 0.03 - 0.03 = 0$
$= \boxed{0\%}$